

AN EXTENDED TECHNICOLOR MODEL WITH QCD-LIKE SYMMETRY BREAKING

by

Thomas Appelquist and Nick Evans

Department Of Physics

Yale University

PO Box 208120

New Haven

CT 06520,USA.

Abstract

We present a one-doublet extended technicolor model, with all fermions in fundamental representations. The bare lagrangian has no explicit mass terms but generates masses through gauge symmetry breaking by purely QCD-like dynamics. The model generates three families of quarks and leptons and can accommodate the observed third family mass spectrum (including a large top mass and light neutrinos). In addition, we show how the model may be extended to incorporate a top color driven top mass without the need for a strong $U(1)$ interaction. We discuss the compatibility of the model with experimental constraints and its possible predictive power with respect to first and second family masses.

1 Introduction

The Higgs model of electroweak symmetry (EWS) breaking is less than satisfying because it offers no understanding of fermion masses and is plagued by a technical hierarchy problem with respect to the Higgs mass. Technicolor models [1] break EWS by the formation of fermion condensates in a strongly interacting theory patterned after QCD. There are no fundamental scalars and therefore no Higgs-mass hierarchy problem. It has been proposed that the fermion (quark and lepton) masses could be generated in technicolor models by extending the gauge sector so that the fermions and the technifermions are unified above the EWS breaking scale. In such extended technicolor (ETC) [2] models, the hierarchy of fermion masses is generated by a hierarchy of breaking scales of the unified gauge group. The problem of the origin of the fermion masses is replaced by the problem of the origin of the ETC symmetry breaking scales.

A number of proposals have been made for the origin of the ETC breaking scales. The ETC symmetries may be broken by including Higgs scalars [3] in appropriate representations of the ETC group. This approach, however, is usually assumed to be a low energy approximation to an even higher scale dynamics since it reintroduces the technical hierarchy problem that technicolor is designed to solve. It has so far not pointed the way to an understanding of fermion mass.

A more audacious explanation of the ETC breaking is that the ETC group(s) break themselves by becoming strong at high scales and forming fermion condensates which are not singlets under the ETC group. This is the tumbling mechanism [4]. It is appealing in its economy, but the desired symmetry breaking patterns require placing the fermions and technifermions in unusual, non-fundamental representations, chosen to achieve the desired breaking pattern. Furthermore, tumbling models have so far relied on speculative most-attractive-channel (MAC) analyses to determine the condensates that form at each scale.

In this paper, we explore an alternative approach to the ETC symmetry breaking scales

which is purely dynamical (no fundamental scalars and no bare mass terms in the lagrangian), which puts fermions only in the fundamental representation, and which employs only QCD-like dynamics. It thus avoids the use of MAC analyses as well as non-fundamental representations. Instead, the breaking pattern (the pattern of quark and lepton masses) is arranged here by the choice of groups into which new fermions are placed and the coupling strengths of these groups. Time will tell whether this holds the key to a deeper understanding of the quark and lepton masses.

Dynamical models with fermions in only the fundamental representation of the gauge groups have also been proposed in refs [5, 6]. However, they generate the light fermion masses by means of couplings to new fermions with mass terms containing the observed mass structure and were intended to demonstrate that flavor dynamics could be separated from EW scale physics in ETC models.

The model presented has one doublet of technifermions and involves Pati-Salam unification [7] at high scales. It gives a relatively small contribution to the electroweak parameter S [8, 9] and gives rise to no pseudo Goldstone bosons at the technicolor scale. Within this model we are able to dynamically generate three family-scales, flavor breaking within each family, a large top mass, and light neutrinos. The dynamics responsible for these features do not generate flavour changing neutral currents (FCNC). FCNCs induced by CKM mixing, the origin of which we do not address here, can be suppressed by small mixing angles or the familiar walking [10] and strong ETC [11] solutions to the problem.

The model as presented contains global symmetries above the highest ETC breaking scale (typically of order 1000TeV) that, when dynamically broken, generate exactly massless, physical Goldstone bosons. They couple to ordinary matter through ETC interactions or the standard model (SM) interactions of their constituent fermions. These interactions are suppressed by the ETC scale and are not visible in current laboratory experiments. Astrophysical constraints [13] from stellar lifetimes do, however, rule out light Goldstones

with SM couplings. We anticipate that yet higher scale unifications than those discussed here may generate masses for these Goldstones which are above the astrophysical constraints.

ETC models that generate the large top mass tend to give rise to contributions to the T [8, 9] parameter that are close to the experimental bound. The T parameter may be reduced in top color assisted technicolor models [14] in which the top mass is generated by a close to critical top self interaction. We show how an alternative model of top color may be included simply in our ETC model. Unlike in the original top color model, the isospin breaking that splits the top and bottom masses is the result of chiral non-abelian color groups rather than a strong $U(1)$ gauge group.

In Section 2, we describe the basic QCD-like mechanism for breaking gauge symmetries. We apply this dynamics in the case of a one-doublet model in Section 3. We discuss both family symmetry breaking leading to different mass scales for each of the three quark-lepton families, and flavor symmetry breaking within each family. Phenomenological aspects of the model are also discussed. In Section 4 we discuss how the model may be extended to include a variation on top color assisted technicolor. In Section 5, we summarize the work and present some conclusions.

2 Gauge Symmetry Breaking With QCD-Like Dynamics

In this section, we describe our breaking mechanism using a simple model in which an $SU(N)$ gauge group is broken to i gauged subgroups and an $SU(j)$ global symmetry group using purely QCD-like dynamics. The driving force is an additional $SU(M)$ gauge interaction which becomes strongly interacting at a scale Λ_M . The model contains the essential dynamics used to break the ETC symmetries in the following sections. There, the $SU(N)$ group will be the ETC group, with quarks, leptons, and technifermions in its fundamental representation. There will also be particles transforming according to the fundamental representation of both the $SU(N)$ and $SU(M)$ groups, which will play an active role in the ETC symmetry breaking. In this section, only the latter particles will be included for simplicity.

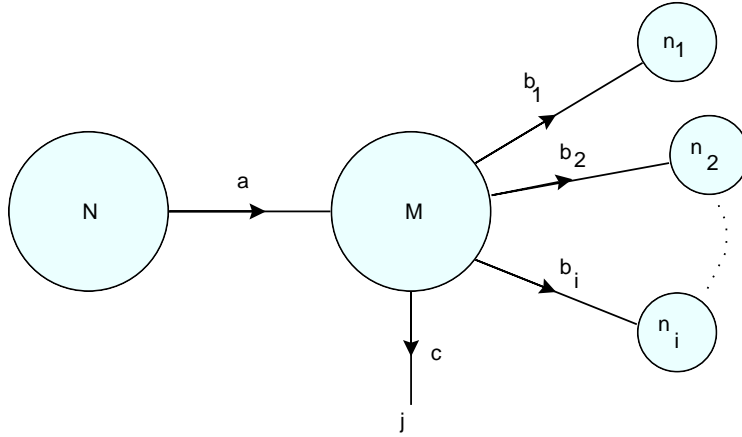


Figure 1: A model of gauge symmetry breaking.

In Fig. 1 we show the model in moose notation [5] with

$$n_1 + n_2 + \dots + n_i + j = N. \quad (2.1)$$

A circled number N corresponds to an $SU(N)$ gauge symmetry and directional lines

represent left-handed Weyl fermions that transform according to the fundamental representation of the gauge groups they connect. A line leaving (entering) a circle with a number N inside represents a fermion transforming under the N (\bar{N}) representation of that group. Lines labelled by a number j that is not circled correspond to j copies of the representation of the gauge group and hence have a global symmetry $SU(j) \otimes U(1)$.

The fermion content of the model pictured in Fig 1 is therefore

	$SU(N)$	$SU(M)$	$SU(n_1)$	$SU(n_2)$	$SU(n_i)$
a	N	\bar{M}	0	0	0
b_1	0	M	n_1	0	0
b_2	0	M	0	n_2	0
$:$	$:$	$:$	$:$	$:$	$:$	$:$
b_i	0	M	0	0	n_i
c	0	M	0	0	0

(2.2)

where the index a runs over the j flavours of the c -fermions. This model is not anomaly free as shown but we shall assume that the additional degrees of freedom required to make $SU(N)$ and the $SU(n_i)$ gauge groups anomaly free do not transform under the $SU(M)$, which is anomaly free with the constraint of Eq. (2.1). The $SU(M)$ will be the only strongly interacting gauge group at its confinement scale Λ_M .

At this scale, the confining $SU(M)$ interaction leads to the formation of the condensates

$$\langle \bar{a}^{1...n_1} b_1 \rangle \neq 0, \quad \langle \bar{a}^{n_1+1...n_1+n_2} b_2 \rangle \neq 0, \quad \dots, \quad \langle \bar{a}^{n_1+n_2+\dots+n_i+1...N} c \rangle \neq 0. \quad (2.3)$$

With the other gauge interactions neglected, the global symmetry on the fermions a , b and c , would be $SU(N)_L \otimes SU(N)_R$. The condensates break this symmetry in the usual pattern

$$SU(N)_L \otimes SU(N)_R \rightarrow SU(N)_V. \quad (2.4)$$

In the presence of the other gauge interactions, the gauged $SU(N)$ group is therefore broken to

$$SU(n_1) \otimes SU(n_2) \otimes \dots \otimes SU(n_i), \quad (2.5)$$

where the gauge field and gauge coupling for each group is a linear combination of the fields and couplings of Fig. 1. We note that all $N^2 - 1$ Goldstone bosons associated with the broken symmetry are eaten by the $N^2 - 1$ gauge bosons that acquire a mass (of order Λ_M).

This symmetry breaking mechanism is of course reminiscent of technicolor itself. Here, as there, the symmetry breaking is driven by an additional, strongly coupled gauge interaction, and the breaking pattern is being imposed by the choice of the $SU(n_i)$ gauge groups. In each case, this is to be compared with the choice of scalar representation in the Higgs mechanism. For ETC breaking, it can also be compared with the choice of fermion representations in tumbling models.

3 One Doublet Technicolor

As an example of ETC breaking using the above mechanism, we construct an ETC model with a single doublet of technifermions, U and D [1, 6]

$$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L, \quad Q_R = \begin{pmatrix} U \\ D \end{pmatrix}_R. \quad (3.1)$$

The quarks and leptons must be unified in a single ETC multiplet with the technifermion doublet. The simplest realization of this unification is a Pati Salam [7] $SU(N+12)$ symmetry where the technicolor group is $SU(N)_{TC}$ and where the SM fermions and technidoublet form the multiplets

$$\begin{aligned} \mathcal{U}_R &= (U, t, \nu_\tau, c, \nu_\mu, u, \nu_e)_R, \\ \Psi_L &= \left(\begin{pmatrix} U \\ D \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu_e \\ e \end{pmatrix} \right)_L, \\ \mathcal{D}_R &= (D, b, \tau, s, \mu, d, e)_R. \end{aligned} \quad (3.2)$$

3.1 Family Structure

3.1.1 A Single Family Model

To introduce the model we restrict attention to the technidoublet and the third family quark and leptons only. The ETC group is then $SU(N+4)$. The model is shown in moose notation in Fig 2.

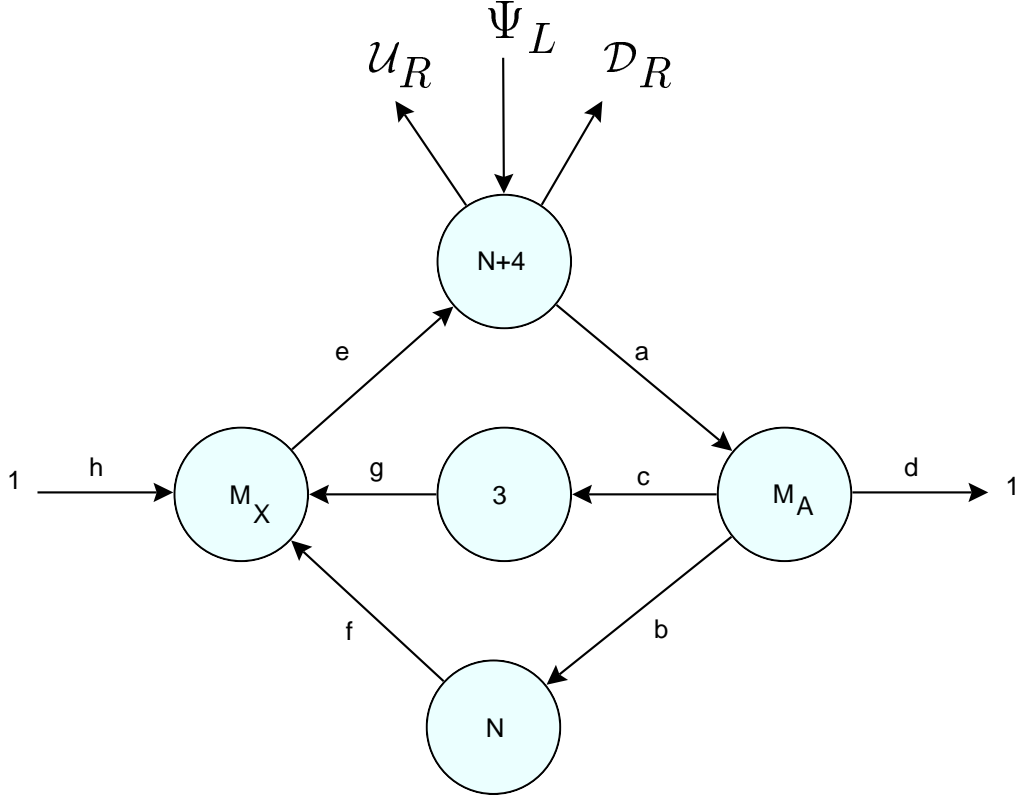


Figure 2: A one family, one technidoublet ETC model .

The $SU(M)$ gauge groups become strong in the order A and then X (at scales Λ_A and Λ_X both of order a few TeV), triggering the breaking of the ETC group to $SU(N)_{TC}$. Consider the highest of these two scales, Λ_A . The fermions transforming under $SU(M)_A$ also transform according to the fundamental representations of the gauged $SU(N+4) \otimes SU(N) \otimes SU(3)$. The $SU(3)$ gauge group is present in order to leave an unbroken $SU(3)$ subgroup of $SU(N+4)$, which will become QCD, acting on the third family of quarks. The strong $SU(M)_A$ interactions form condensates

$$\langle \bar{a}^{1..N} b \rangle \neq 0, \quad \langle \bar{a}^{N+1..N+3} c \rangle \neq 0, \quad \langle \bar{a}^{N+4} d \rangle \neq 0, \quad (3.3)$$

breaking the gauged $SU(N+4) \otimes SU(N) \otimes SU(3)$ symmetry to $SU(N) \otimes SU(3)_{QCD}$. The multiplets in Eq. (3.2) are broken, with the $SU(3)$ subgroup corresponding to the t and b

quarks with QCD interactions, the singlet to the first family lepton doublet and the $SU(N)$ subgroup to the unbroken technicolor gauge group. All $(N + 4)^2 - 1$ Goldstone bosons generated at this first stage of breaking are eaten by gauge bosons which acquire masses of order the confinement scale.

The ETC gauge bosons corresponding to generators broken at Λ_A acquire masses of order F_A , the decay constant of the Goldstone bosons formed at Λ_A that are eaten by the gauge bosons ($F_A^2 \simeq M\Lambda_A^2/4\pi^2$). Below the technicolor scale, where the technifermions condense, these gauge bosons will generate masses for the third family quarks and leptons given by

$$m_f \simeq \frac{\langle \bar{Q}Q \rangle}{F_A^2}, \quad (3.4)$$

where we have assumed that the ETC coupling is perturbative and have used the four fermion approximation for the ETC gauge boson. The ETC gauge boson's mass is proportional to its coupling ($M_{ETC}^2 \simeq g^2 F_A^2$) and hence the ETC coupling cancels in the quark and lepton masses. In this simple model the quarks and leptons are degenerate. We shall address generating flavor breaking within each family in section 3.2.

To cancel anomalies in the model, the additional fermions, e , f , g and h , transforming under the $SU(M)_X$ gauge group have been introduced. The $SU(M)_X$ group confines these new fermions to remove them from the physical spectrum at low energies. We assume that this confinement scale, Λ_X , lies between the technicolor scale and the $SU(M)_A$ confinement scale. At the scale Λ_X there is a global $SU(N + 4)_L \otimes SU(N + 4)_R$ symmetry acting on the fermions transforming under $SU(M)_X$. The preferred vacuum alignment is that no gauge interactions are broken at this extra breaking scale so there are $(N + 4)^2 - 1$ Goldstone bosons which are not eaten. The Goldstone's that transform under the adjoint or fundamental representations of technicolor or QCD acquire masses governed by the scale Λ_X . The remaining two Goldstones are massless and we leave discussion of them to section 3.7.

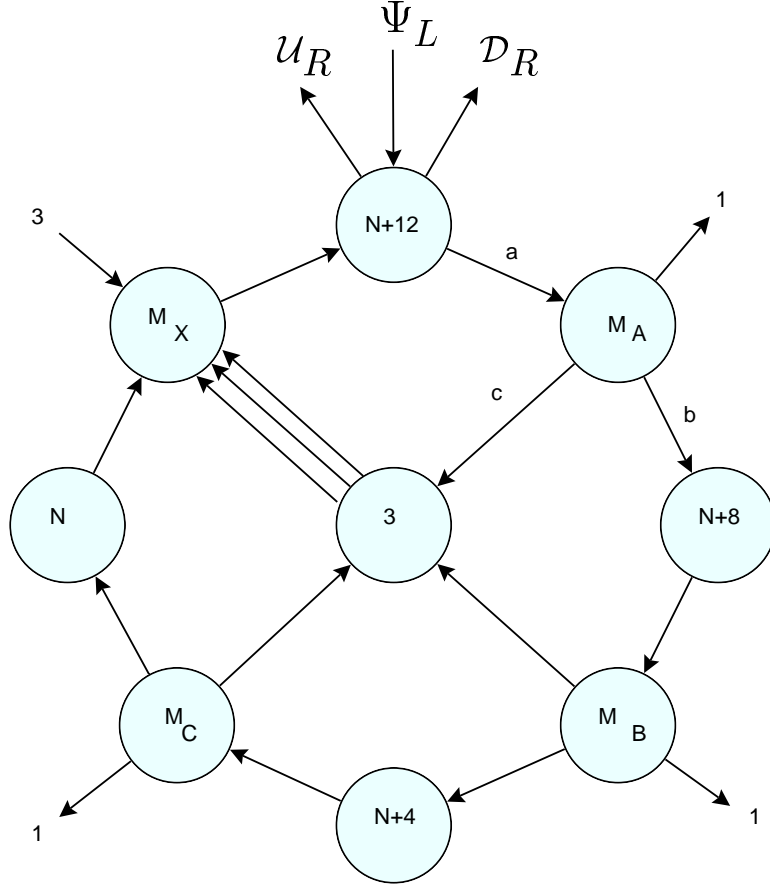


Figure 3: A one doublet ETC model with three family scales.

3.1.2 Three Families

The model can be generalized to include three families of quarks and leptons as shown in Fig 3. The ETC symmetry $SU(N + 12)$ is broken to $SU(N)_{TC} \otimes SU(3)_{QCD}$ at three separate scales. There is a separate $SU(M)$ group to trigger the breaking at each scale. Each is assumed to become strongly interacting in the order A (at a scale of order a few 100's of TeV), B (at a scale of order a few 10's of TeV), and finally C (at a scale of order a few TeV). At each scale the breaking pattern is the same as that discussed in the one family model; at Λ_A the ETC symmetry $SU(N + 12)$ is broken to $SU(N + 8) \otimes SU(3)$. This breaking pattern is then repeated by the groups B and C. At the scale Λ_B it is the $SU(3)$ containing

the $SU(3)$ subgroup of $SU(N+12)$ broken at the scale Λ_A , and an $SU(3)$ subgroup of the $SU(N+8)$ group that break together to an $SU(3)$ group that finally at the lowest breaking scale becomes QCD. The QCD interactions are finally shared by all quarks in the model. The broken gauge bosons of the ETC group now divide into three sets: those with masses of order F_A connecting the first family of SM fermions to more massive generations; those with masses of order F_B connecting the second family to more massive generations; and those with masses of order F_C connecting the third family to technifermions. This hierarchy of ETC gauge bosons masses will generate the hierarchy of quark and lepton family masses below the technicolor scale.

Anomalies are again cancelled in the model by the fermions transforming under the extra $SU(M)_X$ gauge group that confines these fermions between the technicolor and lowest ETC scale. In the enlarged model there are 6 Goldstone bosons that have no gauge interactions and are hence massless.

3.2 Flavor Symmetry Breaking

The model in Fig 3 has an $SU(8)$ flavor symmetry within each family, broken only by the weak SM interactions. To generate the observed quark and lepton masses we must introduce quark-lepton symmetry breaking interactions and isospin symmetry breaking interactions for both the quarks and leptons. For ease of understanding let us discuss a model of just the third family and the technidoublet.

3.2.1 Isospin Breaking

We shall break isospin degeneracy by making the ETC gauge group chiral [15]. We take it to be $SU(N+4)_L \otimes SU(N+4)_{U_R} \otimes SU(N+4)_{D_R}$, as shown in the model in Fig 4. The one family model in Fig 2 is shown by the full lines in Fig 4, with the additional sectors discussed in this section shown as dashed lines.

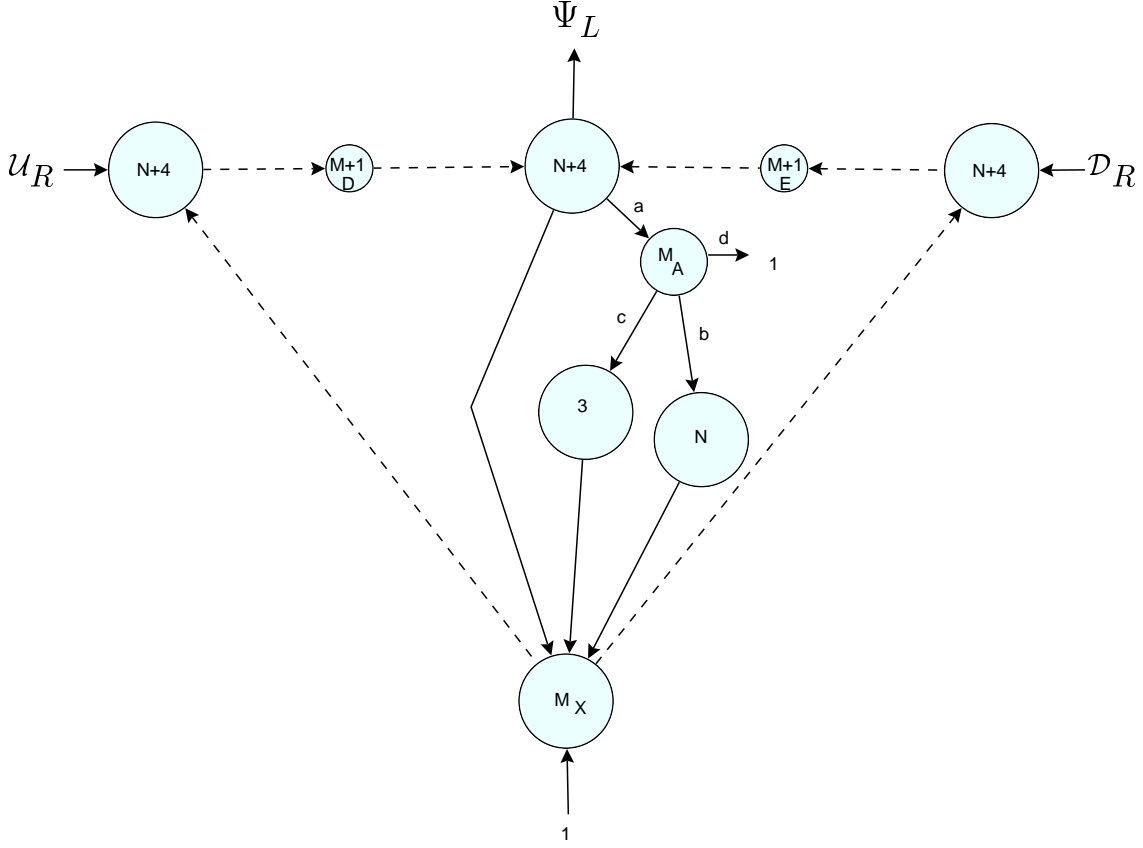


Figure 4: Isospin breaking in the model of the third family.

The $SU(M)_A$ gauge group forms condensates at Λ_A and breaks the $SU(N+4)_L$ ETC group to $SU(N) \otimes SU(3)$ as in the simplest model. The two gauge groups $SU(M+1)_D$ and $SU(M+1)_E$ then become strong between the scale Λ_A and the technicolor scale (for the purposes of making estimates we shall take $\Lambda_A \simeq \Lambda_D \simeq \Lambda_E$), breaking the chiral ETC groups to the vector $SU(N)_{TC} \otimes SU(3)_{QCD}$. At each of these breakings, all Goldstone modes are eaten by gauge bosons associated with broken generators.

There are now three degrees of coupling freedom associated with the interactions of the quarks and leptons: the $SU(N+4)_L$ coupling g_L ; the $SU(N+4)_{\mathcal{U}_R}$ coupling $g_{\mathcal{U}_R}$; and the $SU(N+4)_{\mathcal{D}_R}$ coupling $g_{\mathcal{D}_R}$. The couplings that enter into the quark and lepton masses are these running couplings evaluated at the breaking scale of the ETC interactions and they will in general break the isospin symmetry of the model. The left and right handed ETC gauge

bosons mix through loops of the fermions transforming under $SU(M+1)_D$ and $SU(M+1)_E$ that have condensed at $\Lambda_{D,E}$ as shown in Fig 5. We shall use these extra degrees of freedom to generate the top-bottom mass splitting. The two extra parameters will not be sufficient to explain quark-lepton mass differences which we leave to the next section.

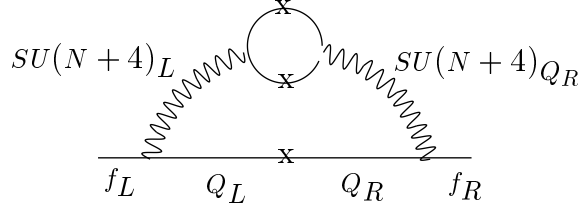


Figure 5: Generation of third family fermion, f , mass from the technifermion, Q , condensate.

If we assume that the ETC gauge bosons coupling to the top have couplings, g_L and g_{U_R} , of order one or greater (but less than 4π at which the ETC gauge bosons become strongly coupled) these gauge bosons will have masses of order $F_A \simeq F_D$ or larger. We may approximate them at the technicolor scale by four fermion interactions. The ETC couplings cancel as in Eq(3.4) and the top mass can be estimated to be roughly

$$m_t \simeq \frac{N}{4\pi^2} \frac{\Sigma(0)^2}{F_A^2} \Sigma(0) \quad (3.5)$$

where $\Sigma(p)$ is the dynamical technifermion mass. A simple Pagels-Stokar [16] estimate, compatible with QCD, gives $v^2 \equiv (250 GeV)^2 \simeq \frac{N}{4\pi^2} \Sigma(0)^2$ and hence $\Sigma(0) \simeq 1 TeV$ for $N \simeq 2$. To generate m_t in the $100 + GeV$ range therefore requires $F_A \simeq 800 GeV$. Although F_A must be close to the technicolor scale, the scale $\Lambda_A \simeq 2\pi F_A / \sqrt{M}$ will be larger, as in QCD, and hence there is some running space for the technicolor coupling to evolve from its value at the breaking scale to its critical value at the technicolor scale. The estimates above are clearly naive approximations to the full non-perturbative technicolor dynamics and are not to be trusted to more than factors of two. It is therefore not completely clear whether a $175 GeV$ top mass may be generated by perturbative ETC gauge bosons.

If the ETC coupling is raised close to its critical value (the value of the ETC coupling at which the ETC interactions alone would break the chiral symmetry of the quark and leptons) at the ETC breaking scale then the approximations above are not valid and the ETC coupling will not cancel from the top mass. A 175GeV top mass may be generated, though how close the ETC coupling must be to its critical coupling is unclear. We shall assume that the ETC interactions are perturbative in the discussions below leaving the possibility that they might be strong and near-critical to section 4.

To generate a smaller bottom mass we take the coupling $g_{\mathcal{D}_R}$ to be less than one. The ETC gauge bosons associated with $SU(N+4)_{\mathcal{D}_R}$ therefore acquire a mass $g_{\mathcal{D}_R}F_E$, which are light relative to $F_{D,E} \simeq F_A \simeq \Sigma(0)$. Referring again to Fig 5, the bottom mass is given approximately by

$$m_b \simeq \frac{N}{4\pi^2} \int dk^2 k^2 \frac{\Sigma(k)}{k^2 + \Sigma(k)^2} \frac{g_{\mathcal{D}_R}^2}{k^2 + g_{\mathcal{D}_R}^2 F_A^2} \quad (3.6)$$

where we have taken $F_E = F_A$ and set the external momentum to zero. With $g_{\mathcal{D}_R}^2 F_A^2 < \Sigma(0)^2$, the integral can be estimated to give roughly.

$$m_b \simeq \frac{N}{4\pi^2} g_{\mathcal{D}_R}^2 \Sigma(0) \quad (3.7)$$

where we have again neglected interactions between the ETC gauge boson and the technicolor gauge bosons. The bottom mass is thus suppressed relative to the top mass by $g_{\mathcal{D}_R}^2$. The choice $g_{\mathcal{D}_R} \simeq 1/6$ gives a realistic value for m_b and leads to a mass of order 200-300GeV for the $SU(N+4)_{\mathcal{D}_R}$ ETC gauge boson.

The technifermion mass splitting $\Delta\Sigma(p) \equiv \Sigma_U(p) - \Sigma_D(p)$ can also be estimated perturbatively in the ETC interactions. The main contribution in the model is from the isospin violating, massive gauge bosons that transform under the adjoint representation of $SU(N)$. The splitting can be estimated to be roughly

$$\Delta\Sigma \simeq \frac{N}{4\pi^2} \frac{\Sigma(0)^3}{F_A^2} \simeq m_t. \quad (3.8)$$

We discuss the implications of this mass splitting for the $\Delta\rho \equiv \alpha T$ parameter in section 3.5 below.

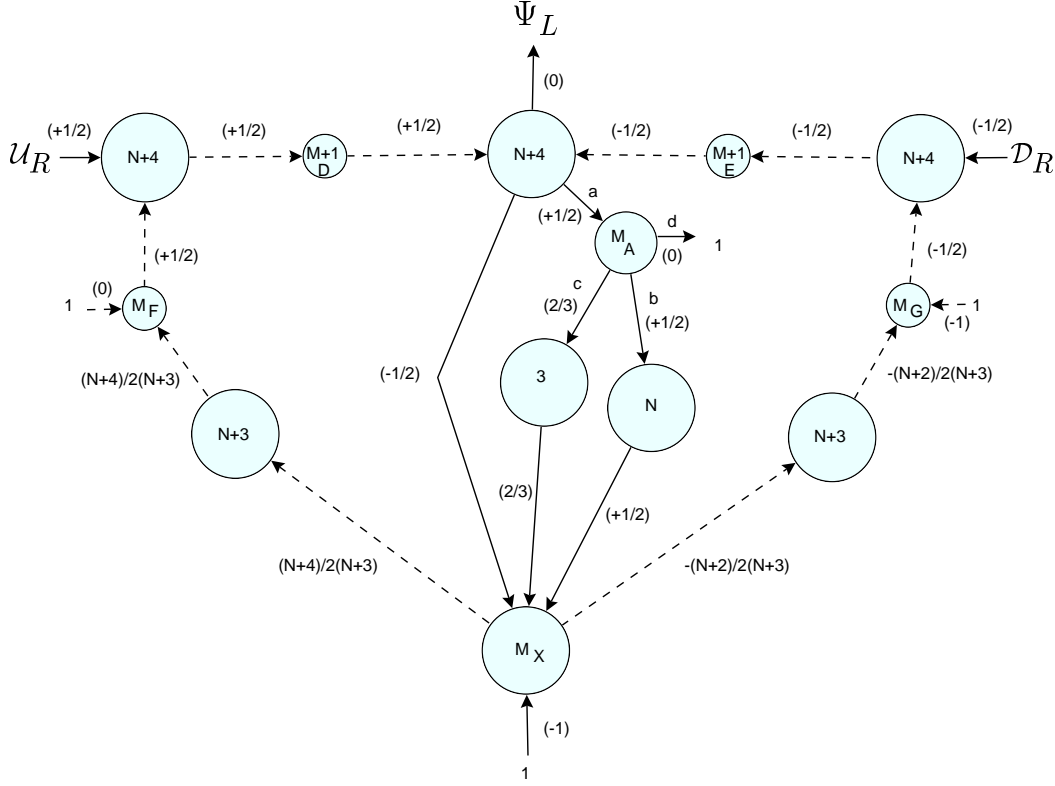


Figure 6: Quark lepton mass splitting in the model of the third family. The fermion lines are labelled by their $U(1)$ hypercharges.

3.2.2 Lepton Masses

In the model in Fig 4, the lepton's interactions are only split from their quark isospin partners by SM interactions. Although QCD interactions may be enhanced if the ETC interactions are close to critical (a possibility we discuss below) and hence could possibly explain the tau-bottom mass splitting they can not explain why the tau neutrino is so light or massless. In order to give a fully perturbative ETC model we shall generate the tau-bottom and tau neutrino-top mass splittings by further ETC breaking dynamics at new scales.

The extra sectors are shown in Fig 6. The $SU(M)_F$ gauge group becomes strongly interacting at the scale Λ_F and breaks a single gauge color from the $SU(N+4)_{\mathcal{U}_R}$ gauge group. The corresponding broken eigenstate of the multiplet in (3.2) will become the neutrino

with mass

$$m_{\nu_\tau} \simeq \frac{N}{4\pi} \frac{F_D^2}{F_A^2} \frac{\Sigma(0)^3}{F_F^2} \quad (3.9)$$

with $F_A \sim F_D$ and with a suitably high choice of F_F ($\geq 100\text{TeV}$) the tau neutrino mass may be suppressed below the experimental bound of roughly 30MeV .

The gauge group $SU(M)_G$ plays the same role for the tau lepton suppressing it's mass relative to the bottom quark's by F_E^2/F_G^2 from which we learn that F_G must be of order a few TeV to reproduce the observed tau-bottom mass splitting.

3.3 The First And Second Families

The lightest two families of quarks and leptons may be incorporated in the model following the discussion in section 3.1.2 and will have mass scales set by the higher two ETC breaking scales. The top-bottom mass splitting will feed down to the lightest two family quarks, generating isospin breaking that could explain the charm-strange mass splitting. The three right handed neutrinos could all be broken from their ETC multiplet at the scale Λ_F . The neutrino masses would then be suppressed relative to the charged lepton masses by $(F_D/F_F)^2$. The single scale Λ_F could thus serve to explain the lightness of all three neutrinos. The quark-lepton mass splittings however can probably not be generated from the third family in perturbative ETC models, since the bottom and tau contributions to, for example, the strange and muon masses are small in comparison with the feeddown from the technifermions' self energies. If necessary extra breaking scales may be introduced to explain the splittings using the dynamics discussed above. Similarly the ETC gauge groups acting on the right handed up and down quarks may be broken at additional scales providing the freedom to accommodate the up-down mass inversion. The symmetry breaking patterns presented here are not capable of producing the CKM mixing angles in the quark sector since the families correspond to distinct ETC gauge eigenstates broken at different scales. We leave the generation of the mixing angles for future work.

3.4 U(1) Embedding

Hypercharge may be embedded in the moose model of Fig 6 by assigning each particle the $U(1)$ charge indicated on the fermionic lines. The final hypercharge group is a subgroup of the $U(1)_R$ group of the quarks, leptons and technifermions and the broken diagonal generators of the $SU(N + 4)$ ETC group. To achieve the correct breaking pattern the condensates formed by $SU(M)_A$ must be invariant to $U(1)_Y$. Since the $SU(N + 4)$ symmetry of the fermions transforming as an \bar{M}_A is explicitly broken their $U(1)$ charges must correspond to the relevant subgroup of their $SU(N + 4) \otimes U(1)$ symmetry.

3.5 Phenomenology

Since there is only one technidoublet in the model there are no pseudo Goldstone bosons generated at the technicolor scale. The single doublet will also generate only a small contribution to the S parameter [8, 9], $S \sim 0.1N$, which we expect to be compatible with the current experimental two standard deviation upper limit $S < 0.4$.

The isospin violating ETC interactions will, of course, give rise to a contribution to the $\Delta\rho(= \alpha T)$ parameter. The W and Z masses are generated by techifermion condensation and deviations from the $\Delta\rho$ parameter from corrections to the relevant diagrams due to exchange of isospin violating ETC gauge bosons. At first order in the ETC interactions the largest contribution will be generated by the exchange of the massive gauge bosons transforming under the adjoint of $SU(N)$ across the techifermion loop. We estimate this “direct” contribution [20] to be

$$\Delta\rho \simeq \frac{v^2}{8F_A^2} \quad (3.10)$$

which is of order a percent.

The isospin violation of the ETC interactions will also feed into the technidoublet giving rise to mass splitting between the techniup and technidown (estimated in Eq(3.8)). There is thus an “indirect” contribution to the $\Delta\rho$ parameter from loops of non-degenerate tech-

nifermions which is second order in ETC gauge boson exchange. Roughly estimating the contribution using the perturbative result for $\Delta\rho$ [8] and the estimate of $\Delta\Sigma$ in Eq(3.8) we find

$$\Delta\rho \simeq \frac{N\Delta\Sigma^2}{12\pi^2v^2} \simeq \frac{v^4}{3F_A^4}. \quad (3.11)$$

These estimates of $\Delta\rho$ are of course naive, ignoring the effects of the strong technicolor dynamics between the technifermion loops and neglecting a complete analysis of the many massive ETC gauge bosons. If they are accurate they could be difficult to reconcile with the experimental constraint $\Delta\rho \lesssim 0.3\%$. We leave a more detailed computation of $\Delta\rho$ to a subsequent paper. In any case, in section 4 below we present a variation of the model that will not overly infect $\Delta\rho$.

The model will also give rise to corrections to the $Zb\bar{b}$ vertex . These arise from both the exchange of the *sideways* gauge boson [17], coupling technifermions to the bottom, across the $Zb\bar{b}$ vertex and from mixing of the Z with the diagonal broken ETC generator [18]. Each of these contributions can be as large as a few percent for an ETC scale of order 1TeV but have opposite signs. The magnitude and sign of the combined correction has been shown to be compatible with the experimental measurement for some models (the exact correction is dependent on N and the relative sizes of g_L and g_{U_R}).

As presented the model does not give rise to quark or lepton number changing FCNCs since each family's quark and lepton number are conserved ETC charges in the model. Of course the most stringent FCNC constraints on ETC models come from $K^0\bar{K}^0$ mixing through the CKM mixing angles which break quark number within each family to a single subgroup. Since we have not addressed the generation of these mixing angles in this paper we can not address this constraint. We note though that these FCNCs may be suppressed in several ways; by small mixing angles in the up-type quark sector, or by a walking technicolor theory or strong ETC interactions that enhance the ETC scales.

3.6 Massless Goldstone Bosons

Massless Goldstone bosons are generated in the model at the scale Λ_X as discussed above. These Goldstones carry no charge under any of the gauge groups in the model. However, their constituents are charged, so they can be produced by gluon or photon fusion or in the decay of the Z [12]. They can also be produced through the exchange of the heavy ETC gauge bosons. The amplitude in each case is proportional to $1/F_X$ where $F_X \simeq 1\text{TeV}$, so that the production rate is down by at least an order of magnitude relative to the production of the Goldstones composed of technifermions that arise in a one family technicolor model. The rate is below current laboratory limits. With the Goldstones massless or very light, however, their production by the above mechanisms is a major energy loss mechanism for stars [13], and is ruled out by stellar abundances.

The Goldstones are thus troublesome but may acquire masses from further unifications above the scales discussed in the model so far. In the spontaneous breaking at Λ_X , the Goldstone bosons complete an adjoint representation of the unbroken $SU(N+12)$ vector global symmetry group (in the three family model). If at some higher scale this group is gauged (corresponding for example to gauging the full chiral symmetry group in Fig 3) then all the Goldstones will acquire masses given by

$$M_A^2 \simeq 4\pi F_X^4 / \Lambda_{new}^2 \quad (3.12)$$

which is potentially sufficient to ensure that the Goldstones will not be a source of energy loss in stellar interiors.

4 Strong ETC and Chiral Top Color

The model presented so far appears capable of producing a 175GeV top mass treating the ETC interactions perturbatively without contradicting other experimental bounds. However, the contributions of the isospin violating ETC gauge bosons to the $\Delta\rho$ parameter and to the

$Zb\bar{b}$ vertex are close to experimental limits. These contributions, which scale as $1/M_{ETC}^2$, can be reduced by increasing the lowest ETC scale, but at the expense of tuning the ETC coupling close to its critical value from below to generate the large top mass. A near critical ETC interaction for the third family would also enhance the QCD corrections to the third family quark masses and potentially explain the bottom tau mass splitting without the need for the extra ETC symmetry breaking scale Λ_G discussed in section 3.2.2. Finally increasing the lowest ETC scale would allow us to increase the scale Λ_X and hence generate larger masses for the Goldstones formed at that scale.

Although near critical ETC interactions at a higher ETC scale may suppress the direct contribution to $\Delta\rho$ the indirect contribution will remain roughly the same size but may no longer be considered second order. This follows from a gap equation analysis which suggests that the technifermion mass splitting will remain of order m_t . Therefore if the large top mass is the result of either perturbative or strongly interacting sideways ETC interactions the contribution to the $\Delta\rho$ parameter may conflict with the experimental limit.

Recently Hill [14] has proposed that the large top mass may be generated by a near critical top self interaction [19]. If the ETC gauge boson with the large isospin violating coupling responsible for the top mass does not couple to the technifermions then the isospin splitting will not feed back into the technisector and hence the $\Delta\rho$ parameter as described above. Hill generates the top self interaction by assuming that at ETC scales there is a separate $SU(3)_C \otimes U(1)_Y$ gauge group acting on the third family that is near critical when broken to the SM gauge groups.

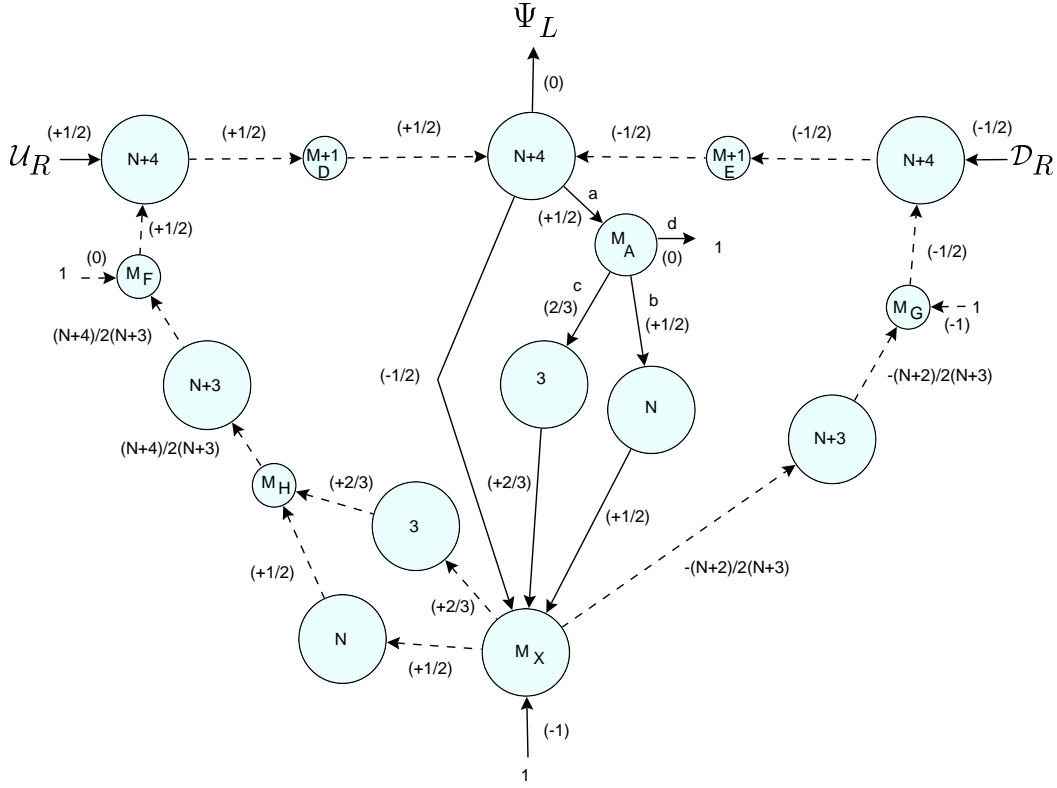


Figure 7: Chiral top color in the model of the third family.

We can extend our model to include a top color interaction as shown in Fig 7. The new $SU(M)_H$ group becomes strongly interacting at Λ_H breaking the $SU(N+3)_{\mathcal{U}_R}$ group, left after the right handed neutrino has decoupled, to $SU(N) \otimes SU(3)$. The right handed $SU(3)$ color group's coupling will run independently of the technicolor coupling below this breaking scale (we require that Λ_H is large enough that there is enough running time for the $SU(3)$ and $SU(N)$ groups' couplings to significantly diverge) and this interaction of the top will be assumed to be near critical when broken to the vector QCD subgroup at Λ_D . Unlike in Hill's model in which the top bottom mass splitting is the result of a strongly coupled $U(1)_Y$ gauge interaction (with the associated problem of it's coupling being close to it's Landau pole) here the isospin splitting is provided by chiral, asymptotically free, *non-abelian* gauge groups.

5 Summary and Conclusions

We have presented a one-doublet technicolor model in which the ETC gauge symmetries are broken by purely QCD-like dynamics. All fermions transform only under the fundamental representation of gauge groups. The model has chiral ETC gauge groups, explicitly breaking custodial symmetry, and Pati-Salam unification at high scales. It's main features are:

- Three families of quarks and leptons are incorporated, with a hierarchy of three family-symmetry breaking scales. Within the third family, the full spectrum of masses can be accommodated. In particular, we argue that with a third family ETC scale on the order of 1TeV , it may be possible to generate both the top and bottom masses through perturbative ETC interactions. A light tau neutrino mass can be achieved by breaking the ETC group for right handed isospin $+1/2$ fermions at a high scale. To place m_{ν_τ} below the current limit of roughly 30MeV , this scale must be above about 100TeV .
- Since the model contains a single doublet of technifermions, no pseudo-Goldstone bosons are formed at the electroweak scale and the S parameter can be kept relatively small. The weak custodial isospin symmetry breaking built into the model leads to a so called “direct” contribution [20] to $\Delta\rho \equiv \alpha T$, which is first order in the ETC interaction. Our naive estimate suggests that this contribution may be nearly 1% and hence possibly above the experimental limit. A more detailed analysis of this contribution (and that to the $Zb\bar{b}$ vertex) will be given in a succeeding paper. The “indirect” contribution, arising from loops of non-degenerate technifermions, is second order in ETC interactions and is small relative to the direct contribution when the ETC interactions are perturbative.
- The model contains global symmetries at the ETC scales, whose spontaneous breaking leads to massless Goldstone bosons. They can couple to ordinary matter through SM interactions and are ruled out by stellar energy loss constraints [13]. They can,

however, be given phenomenologically acceptable masses by further unifications above the ETC scales, which break the global symmetries.

- Some of the mass splittings within the first two families will be fed down naturally from the third family. We have argued that the charm strange mass splitting may be a result of the top bottom mass splitting. The suppression of all three generations of neutrino masses may be explained by a single ETC breaking scale. We have not discussed the origin of quark mixing angles in this work though it will clearly be important to address this point in the future.
- We have also demonstrated that a large top quark mass can be generated dynamically in technicolor by a near critical top color interaction without the need for a strong U(1) interaction. This variant of the model is compatible with the experimental value of $\Delta\rho$.

The model presented here illustrates that ETC symmetries can be broken using only QCD-like dynamics and fermions in fundamental representations. The requisite number of quark-lepton and isospin symmetry violating parameters may be introduced to accommodate the third family spectrum. It remains to be seen whether this approach leads to an explanation of quark and lepton masses and CKM mixing angles.

Acknowledgements

The authors would like to thank Steve Hsu, Steve Selipsky, Andy Cohen, Sekhar Chivukula, Ken Lane, Liz Simmons and Lisa Randall for useful comments and discussion.

References

- [1] E. Farhi and L. Susskind, Phys. Report 74 No.3 (1981) 277.
- [2] S. Dimopolous and L. Susskind, Nucl. Phys. **B155** (1979) 237; E. Eichten and K. Lane, Phys. Lett. **B90** (1980) 125.
- [3] S.F. King, Phys Rev **D46** (1992) 4097; S.F. King and S. Mannan, Nucl Phys **B369** (1992) 119
- [4] S. Dimopolous, S. Raby and L. Susskind, Nucl. Phys. **B169** (1980) 373; T.W. Appelquist and J. Terning, Phys. Rev. **D50** (1994) 2116.
- [5] H. Georgi, “Technicolor and Families”, Proc. 1990 International Workshop On Strong Coupling Gauge Theories And Beyond, (ed. T. Muta and K. Yamawaki, publ. World Scientific, Singapore,1991); H. Georgi, Nucl Phys **B416** (1994) 699; R.S. Chivukula, Phys. Lett. **B99** (1987) 188.
- [6] L. Randall, Nucl Phys **B403** (1993) 122.
- [7] J.C. Pati and A.Salam, Phys. Rev. Lett. **31** (1973) 661; Phys. Rev. **D8** (1978) 1240.
- [8] M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65** (1990) 964; M.E. Peskin and T. Takeuchi, Phys. Rev. **D46** (1992) 381.
- [9] C.P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, Phys. Lett. **B326** (1994) 276; I. Maksymyk, C.P. Burgess and D. London, Phys. Rev. **D 50** (1994) 529; P. Bamert and C.P. Burgess, McGill preprint, McGill-94/27 hep-ph 9407203.
- [10] B. Holdom, Phys. Rev. **D 24** (1981) 1441; B. Holdom, Phys. Lett. **B150** (1985) 301; T.W. Appelquist and L.C.R. Wijewardhana, Phys. Rev. **D36** (1987) 568.
- [11] T.W. Appelquist and L.C.R. Wijewardhana, Phys. Rev. **D36** (1987) 568; S.F. King and D.A. Ross, Phys. Lett. **B3** (1989) 363; T. Appelquist, T. Takeuchi, M. Einhorn, and

- L.C.R. Wijewardhana, Phys. Lett. **B220** (1989) 223; T. Appelquist and O. Shapira, Phys. Lett. **B249** (1990) 83.
- [12] V. Lubicz, Nucl. Phys. **B404** (1993) 559, and references contained therein.
- [13] J.E. Kim, Phys rep **150** (1987) 1; M.S. Turner, Phys. Rep. **197** (1990) 67
- [14] C.T. Hill, Phys. Lett. **B345** (1995) 483; R.S. Chivukula, B.A. Dobrescu and J. Terning, Boston University preprint BUHEP-95-08, hep-ph 9503203.
- [15] M.B. Einhorn and D. Nash, Nucl. Phys. **B371** (1992) 32.
- [16] H. Pagels and S. Stokar, Phys. Rev. **D20** (1979) 2947.
- [17] R.S. Chivukula, S.B. Selipsky and E.H. Simmons, Phys. Rev. Lett. **69** (1992) 575; R.S. Chivukula, E. Gates, E.H. Simmons and J. Terning, Phys. Lett. **B311** (1993) 157; N. Evans, Phys. Lett. **B331** (1994) 378.
- [18] B. Holdom Phys. Lett. **B339** (1994) 114; G. Wu, Phys. Rev. Lett. **74** (1995) 4137.
- [19] Y. Nambu, “New Theories In Physics”, Proc. XI Warsaw Symposium on Elementary Particle Physics, (ed. Z. Adjuk *et al.*, publ. World Scientific, Singapore, 1989); V.A. Miransky, M. Tanabashi and M. Yamawaki, Phys. Lett. **B221** (1989) 177; W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. **D41** (1990) 1647.
- [20] T.W. Appelquist, M.J. Bowick, E. Cohler, A.I. Hauser, Phys. rev. **D31** (1985) 1676.